Acknowledgments

The research reported in this article is sponsored by the Defense Modeling and Simulation Office under the Department of Defense and administered by the Naval Air Warfare Center Training Systems Division, Contract N61339-92-C-0110.

References

[1]Lin, K. C., "Dead-Reckoning in Distributed Interactive Simulation," *Proceedings of the 3rd Annual Conference of the Chinese-American Scholars Association of Florida*, Miami, FL, 1992, pp. 1–5.

[2]Hoffman, L. L., and Lin, K. C., "An Experimental Design for Detecting Optimal Simulated Flight Performance on Distributed Interactive Systems," Inst. for Simulation and Training, Univ. of Central Florida, TR IST-CR-93-16, Orlando, FL, May 1993.

Vortex Burst Model for the Vortex Lattice Method

Steven A. Brandt*
U.S. Air Force Academy,
Colorado Springs, Colorado 80840

Introduction

RECENT flight testing of aircraft designed to maneuver at very high angles of attack suggest that such capabilities may be useful in future fighter aircraft. Inexpensive computational tools that accurately predict forces and moments at high angles of attack may be useful for the conceptual design of such aircraft. The vortex lattice method (VLM) has long been used as an analysis tool for attached flow cases. More recently,[1,2] VLM has been used with empirical vortex burst data to give good predictions of trim requirements at higher angles of attack. In the current work, a vortex core model is analytically derived in a form that allows it to be easily incorporated into the VLM. Vortex burst locations predicted by the coupled vortex burst model/vortex lattice method (VBM/VLM) for several swept wings at high angles of attack are in good agreement with empirical data. This makes it possible to substitute model-predicted burst points for the empirical data of Refs. 1 and 2.

Formulation

A model for the vortex core is derived from the steady, incompressible Navier–Stokes equations written for a cylindrical coordinate system centered on the vortex core. The derivation follows, in general, the approaches developed by Mager[3] and Krause[4] and extends the method described in Refs. 5 and 6. The vortex is assumed to be slender and axisymmetric. The u, v, and w velocities are defined in the x, r, and θ cylindrical coordinate system aligned with the vortex central axis. The following variables are defined:

$$\bar{x} = \frac{x}{L} \qquad \varepsilon = \frac{\delta_{\rm ref}}{L} \qquad \bar{r} = \frac{r}{\varepsilon L} = \frac{r}{\delta_{\rm ref}} \qquad Re = \frac{\rho U_\infty L}{\mu}$$

$$\bar{p} = \frac{2p}{\rho U_\infty^2} \qquad \bar{u} = \frac{u}{U_\infty} \qquad \bar{v} = \frac{v}{\varepsilon U_\infty} \qquad \bar{w} = \frac{w}{U_\infty}$$

where L is the length of the vortex to the wing trailing edge, $\delta_{\rm ref}$ is the vortex core radius at $x = L$, and ρ, μ, and U_∞ are freestream density, viscosity, and velocity, respectively. The overbars denote dimensionless quantities, but will be omitted in the subsequent analysis.

As in Ref. 6, the ratio of the vortex core diameter to the vortex length is assumed to be of the order of the inverse of the square root of the Reynolds number. After eliminating terms that become negligible for large Reynolds numbers, the nondimensional equations of motion become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{v}{r} = 0 \tag{1a}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \frac{1}{Re\varepsilon^2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{1b}$$

$$\frac{w^2}{r} = \frac{\partial p}{\partial r} \tag{1c}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial r} + \frac{vw}{r} = +\frac{1}{Re\varepsilon^2}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(rw)}{\partial r}\right] \tag{1d}$$

Algebraic profiles for radial variation of axial and circumferential velocity are chosen and Eq. (1c) is integrated from $r = 0$ to $r = \delta$. For physically reasonable choices of velocity profiles, the integrated form of Eq. (1c), after negligible terms are discarded, is

$$p(x, 0) = p(x, \delta) - K_a(\Gamma^2/a) \tag{2}$$

where

$$a = \pi\delta^2 \quad \text{and} \quad \Gamma = 2\pi\delta w_\delta$$

The subscript δ denotes quantities at the core edge and K_a depends on the choice of the w velocity profile.

The previously chosen u profile is next used in Eq. (1a) to determine the corresponding v profile. Then, Eq. (1d) is integrated over the area of the vortex core to obtain

$$\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}x} = \frac{K_1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}x} - \frac{K_2}{u_\delta}\frac{\mathrm{d}u_\delta}{\mathrm{d}x} + \frac{K_3}{Re\varepsilon^2 a u_\delta} \tag{3}$$

where the values of K_1, K_2, and K_3 depend on the velocity profiles chosen. If a simple solid body rotation is used for the w profile, K_2 and K_3 are zero and $K_1 = 1$, and so Eq. (3) can be integrated with respect to x to yield

$$a \propto \Gamma$$

If more complex w profiles are chosen, the expression for a can still be approximated as

$$a = K_0\frac{\Gamma^{K_1}(x - x_0)}{u_\delta^{K_2}} \tag{4}$$

where K_0 depends primarily on the product $Re\varepsilon^2$. Assuming that the core circumferential velocity profile is sufficiently

Received Oct. 4, 1993; presented as Paper 94-0074 at the AIAA 32nd Aerospace Sciences Meeting and Exhibit, Reno, NV, Jan. 10–13, 1994; revision received Dec. 22, 1994; accepted for publication May 20, 1995.

*Associate Professor of Aeronautics, Department of Aeronautics, 2354 Fairchild Drive, Suite 6H24. Member AIAA.

close to a solid body rotation, the radial pressure distribution can be approximated as

$$p(x, r) = p(x, 0) + (r/\delta)^2[p(x, \delta) - p(x, 0)] \qquad (5)$$

Using Eqs. (2) and (5) one can integrate Eq. (1b) over the area of the core to obtain

$$u_0 \frac{du_0}{dx} + \frac{dp_0}{dx} = -K_a \left(\frac{\Gamma}{a}\frac{d\Gamma}{dx} - \frac{\Gamma^2}{a^2}\frac{da}{dx}\right) \qquad (6)$$

where the subscript 0 denotes quantities at the center of the core.

For the present work, the simple case of a flat-plate delta wing with a conical flow is considered. For these assumptions the vortex strength increases linearly with the core length and the core area increases with the square of the axial distance from the origin. In effect, Eq. (6) describes an additional axial pressure gradient that arises from the growth of the core and strengthening of the vortex. This axial pressure gradient is modified by any pressure gradient that exists in the external pressure field, as it is transmitted to the core via the core radial pressure balance. When bursting occurs, the pressure gradient due to the core growth, with the possible help or hindrance of the pressure gradient in the external pressure field, acts to bring the axial flow in the core to rest. For the purpose of integrating the model into the VLM, the initial flow velocity at the origin of the vortex, as well as the external pressure field along the length of the vortex at the edge of the rotational core, are obtained from the VLM solution. The core-edge pressure field is modeled as a linear scaled image of the wing surface pressure field to avoid the necessity of explicitly locating the vortex core. The flow quantities from the VLM solution are used, along with the expression for core area given by Eq. (4), to evaluate the integral of Eq. (6) with respect to x to yield

$$\frac{u_{0_2}^2 - u_{0_1}^2}{2} + p_{0_2} - p_{0_1} = K_b \Gamma_t^2 \ln \frac{x_2}{x_1} \qquad (7)$$

where the coefficient K_b collects various constants from Eq. (6), from the conical flow assumption, and from the scaling of the external pressure field. Γ_t represents the strength of the vortex at the nondimensional $x = L$, and the subscripts 1 and 2 denote flow quantities at the vortex origin and the point of interest, respectively. A singularity results if the origin of the x, r, and θ coordinate system coincides with the vortex origin. For convenience, this coordinate system is located such that $\ln(1/x_1) = 1$. This choice affects the value of K_b in the integration. At the point in the vortex where bursting occurs, and hence, $u_0 = 0$, Eq. (7) simplifies to

$$(u_{0_1}^2/2) + p_{0_{\text{burst}}} - p_{0_1} = K_b \Gamma_1^2 \ln(x_{\text{burst}}/x_1) \qquad (8)$$

In practice, both sides of Eq. (8) are evaluated for the x coordinate of each vortex panel in the VLM solution. If the right-hand side (RHS) of Eq. (8) is greater than or equal to the left-hand side, then bursting is determined to have occurred at or upstream of that x location. The x coordinate furthest upstream of the panels identified in this way is then taken as the point of vortex bursting. In order to evaluate the RHS of Eq. (8) it is necessary to determine the vortex strength. This was done by using the total circulation on the wing in the VLM solution, assuming that all of the circulation is shed and rolled up into the vortex. This relationship, when combined with Eq. (8) and simplified, using the fact that total circulation in the VLM solution is proportional to total lift, yields

$$\Delta C_{p_{\text{burst}}} = K_f C_L^2 \ln(x_{\text{burst}}/x_1) \qquad (9)$$

where C_L is the wing lift coefficient and K_f is the final single control constant for the vortex model, representing all of the previous integration and control constants. Choosing a particular value for K_f is, in part, choosing a circumferential velocity profile for the vortex model, although many different velocity profiles can produce the same value of K_f. Larger values of K_f result in the onset of bursting at lower angles of attack and more rapid forward progression of the burst point with increasing α.

Results

Although the implementation of the model just described is very simple and computationally inexpensive, the results obtained to date are very encouraging. The particular VLM code that has been used to test the VBM is described in Ref. 7. The VBM/VLM has been tested on two different delta wings and one highly swept tapered wing. The first delta wing (Fig. 1a) was a simple 60-deg true delta. The swept wing (Fig. 1b) had a 60-deg leading-edge sweep angle and an AR of 3.14. The wingtip of the swept wing was cut back at a 45-deg angle to form a 90-deg angle with the trailing edge of the wing. The third wing (Fig. 1c) was a 70-deg true delta.

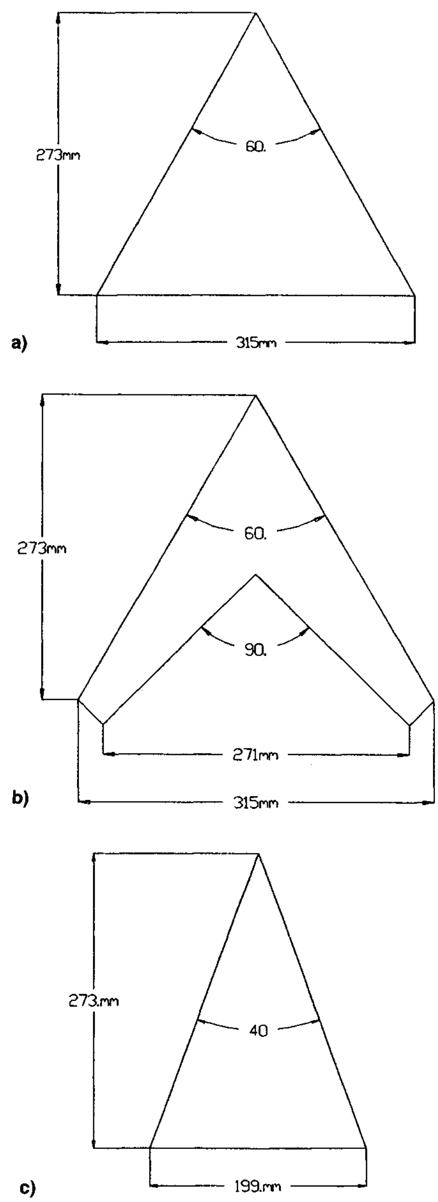

Fig. 1 Wing model geometries.

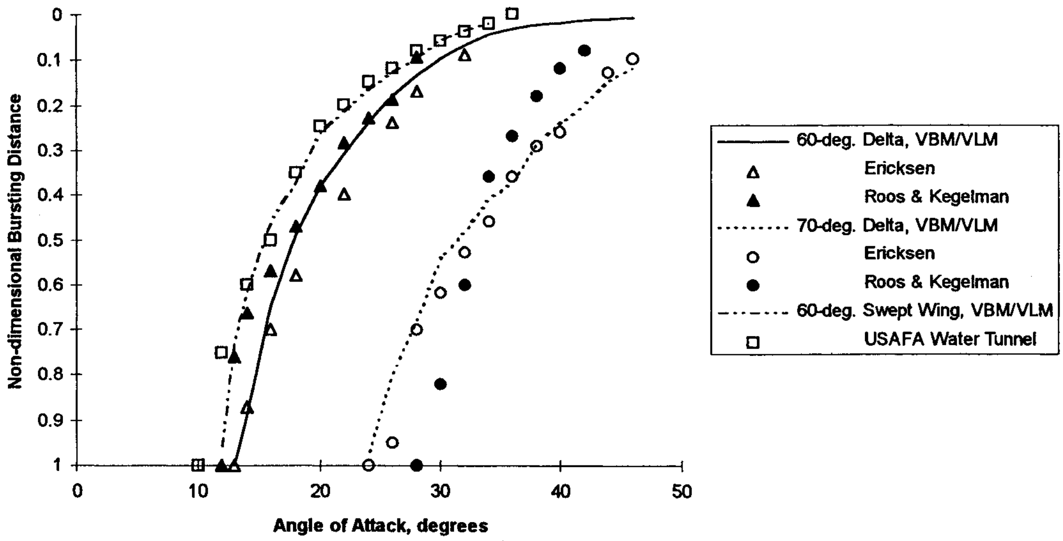

Fig. 2 Vortex burst trajectories.

The control parameter K_f in Eq. (9) is the only variable needed to fine-tune the vortex-burst predictions. It was adjusted for the 60-deg delta wing to achieve onset of vortex burst at the wing trailing edge at $\alpha = 13$ deg. A value of 2.6 for K_f gave this result. When this same value of K_f was used for VBM/VLM computations over a range of angles of attack for each of the wing shapes tested, the vortex burst locations shown in Fig. 1 resulted. Wind-tunnel values for burst locations on similar models from Refs. 8 and 9 are plotted in Fig. 2 for comparison. Note the good agreement between the model and wind-tunnel data for all shapes tested.

Conclusions

A model for leading-edge vortex bursting has been developed and used with the vortex lattice method to predict vortex bursting on highly swept wings. The model was derived from the steady, incompressible Navier–Stokes equations for the vortex core. The method was tested on four highly swept wing models. In all cases vortex burst predictions agreed well with the vortex burst locations observed in wind- and water-tunnel tests.

Acknowledgments

The author greatly appreciates the assistance and advice of Paul Gelhausen, J. R. Gloudemans, and Dave Kinney of NASA Ames Research Center. Without their insight, encouragement, and programming skills the analysis-tool described in this article would still be just an idea.

References

[1]Lan, C. E., "VORSTAB—A Computer Program for Calculating Lateral-Directional Stability Derivatives with Vortex Flow Effect," NASA CR-172501, Jan. 1985.

[2]Lan, C. E., Emdad, H., Chin, S., Sundaram, P., and Mehrotra, S. C., "Calculation of High Angle-of-Attack Aerodynamics of Fighter Configurations," AIAA Paper 89-2188, July 1989.

[3]Mager, A., "Dissipation and Breakdown of a Wingtip Vortex," *Journal of Fluid Mechanics*, Vol. 22, Pt. 4, 1972, pp. 609–628.

[4]Krause, E., "A Contribution to the Problem of Vortex Breakdown," *Computers and Fluids*, Vol. 13, No. 3, 1985, pp. 375–381.

[5]Brandt, S. A., "Numerical Simulation of Leading-Edge Vortex Rollup and Bursting," Ph.D. Dissertation, Univ. of Illinois at Urbana-Champaign, IL, 1988.

[6]Lee, K. D., and Brandt, S. A., "Modeling of Vortex Dominated Flow Fields in the Euler Formulation," *Proceedings of the 16th ICAS Congress*, Jerusalem, Israel, 1988, pp. 1437–1450 (Paper 88-5.9.3).

[7]Miranda, L. R., Elliott, R. D., and Baker, W. M., "A Generalized Vortex Lattice Method for Subsonic and Supersonic Applications," NASA CR-2865, 1977.

[8]Erickson, G. E., "Flow Studies of Slender Wing Vortices," AIAA Paper 80-0098, Jan. 1980.

[9]Wentz, W. H., and Kohlman, D. L., "Vortex Breakdown on Slender Sharp-Edged Wings," *Journal of Aircraft*, Vol. 8, No. 3, 1971, pp. 156–161.

Kill Probability in Antiaircraft Firing Theory

Marek Radomski*
Warsaw University of Technology, 02-524 Warsaw, Poland

Nomenclature

$P_r(x_1, x_2, x_3)$ = damage function[1] (herein after referred to as DF), which determines the value of probability of killing the target on the condition that the center of inertia of the target is in a defined location in space (x_1, x_2, x_3) related to the projectile blast epicenter.

P_r = the probability of killing the target when at least a single hit is scored. It is the average value of DF for the points (x_1, x_2, x_3) contained within the target.

$\boldsymbol{R}_b$ = random vector process or random vector variable that describes ballistic dispersion. It characterizes the location of a projectile relatively to the average trajectory.

$\boldsymbol{R}_t$ = random vector process or random vector variable that describes aiming error. This vector is characterized by a minimum length during the projectile flight and is equal to the difference between the radius vector $\boldsymbol{x}_p$ of the point that overlaps the

Received Oct. 11, 1994; revision received May 8, 1995; accepted for publication May 10, 1995.

*Senior Lecturer, Institute of Mechanics and Design, Narbutta 85.